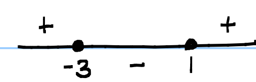
Note Title 12/06/2012

Answers - Additional Practice Understanding Graphs

1) $$f(x) = \begin{cases} x-39 \,; & x \le 4 \\ x^3+3x^2-9x \,; & -4 < x < 3 \\ 30-x \,; & x \geq 3 \end{cases}$$

Min values where $f'(x)$ changes from neg to pos or at endpoint.

$$f'(x) = \begin{cases} 1 \,; & x \le -4 \\ 3x^2+6x-9 \,; & -4 < x < 3 \\ -1 \,; & x \geq 3 \end{cases}$$



$f'(x)=0$ when $3x^2+6x-9=0$

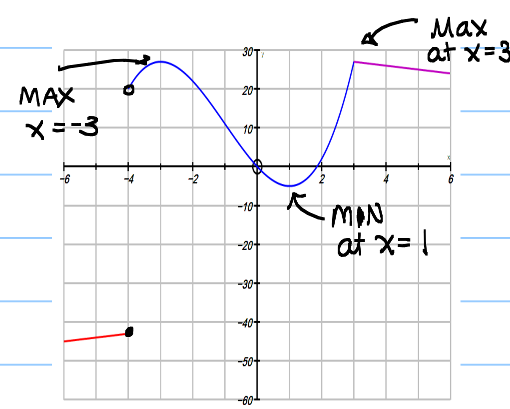
$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3 \quad x=1$

$\therefore$ Min at $x=1$

Max at $x=-3$



2 a) Critical point(s) where $f'(x)=0$ and where $f'(x)$ DNE

$\therefore$ At $x=2$, $x=4$, $x=6.6$ and $x=9$

b) Increasing where $f'(x)>0$

$(0,2) \cup (4,6.6) \cup (9,10)$

c) Local max where $f'(x)$ changes from ⊕ to ⊖

$\therefore$ At $x=2$, $x=6.6$

d) Concave up where $f''(x)>0$

$\therefore$ $(3,6) \cup (7,10)$

3) $g(x) = \begin{cases} -x^3+5x^2-6x \,;\ 0 \le x \le 3 \\ x^2-8x+15 \,;\ x>3 \end{cases}$

$g'(x) = \begin{cases} -3x^2+10x-6 \,;\ 0 \le x \le 3 \\ 2x-8 \,;\ x>3 \end{cases}$ $\qquad g''(x) = \begin{cases} -6x+10 \,;\ 0 \le x \le 3 \\ 2 \,;\ x>3 \end{cases}$

a) Critical point(s) where $g'(x)=0$ and $g'(x)$ DNE

$-3x^2+10x-6=0$

$x = \dfrac{-10 \pm \sqrt{100-4(-3)(-6)}}{2(-3)}$

$= \dfrac{-10 \pm \sqrt{28}}{-6} = \dfrac{-10 \pm 2\sqrt{7}}{-6}$

$x = \dfrac{5}{3} \pm \dfrac{\sqrt{7}}{3}$ ← Both in domain $0 \le x \le 3$

$2x-8=0$

$x=4$ ← in domain $x>3$

Check if $g'(3)$ exists.

$\lim_{x\to 3^-} g'(x) = -3$

$\lim_{x\to 3^+} g'(x) = -2$

∴ Also critical pt at $x=3$

b) $g(x)$ is decreasing when $g'(x)<0$ ∴ for

$\left(\dfrac{5-\sqrt{7}}{3}, \dfrac{5+\sqrt{7}}{3}\right) \cup (4, \infty)$

c) $g(x)$ has min when $g'(x)$ changes from ⊖ to ⊕

at $x = \dfrac{5-\sqrt{7}}{2}$ and $x=4$

d) $g(x)$ is concave down when

$g''(x)<0$ when $-6x+10<0$

$10 < 6x$

$10/6 < x$

∴ $(5/3, 3]$

4) $xy^2 + x = 1$

$1y^2 + x\left(2y\frac{dy}{dx}\right) + 1 = 0$

$2xy\frac{dy}{dx} = -1 - y^2$

$$\boxed{\frac{dy}{dx} = \frac{-1-y^2}{2xy}}$$

$$\left.\frac{dy}{dx}\right|_{\substack{x=1/2 \\ y=1}} = \frac{-1-1}{2(\frac{1}{2})(1)}$$

$= -2$

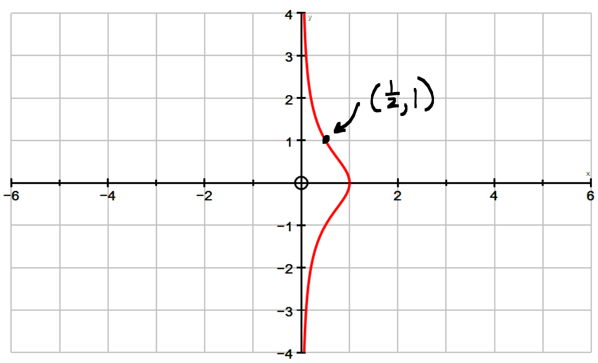
$$\frac{d^2y}{dx^2} = \frac{\left(-2y\frac{dy}{dx}\right)(2xy) - (-1-y^2)\left(2y + 2x\frac{dy}{dx}\right)}{(2xy)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-4xy^2y' + (1+y^2)[2y + 2xy']}{4x^2y^2}$$

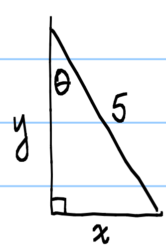
$$\left.\frac{d^2y}{dx^2}\right|_{\substack{x=1/2 \\ y=1}} = \frac{-4(\frac{1}{2})(1)^2(-2) + (1+1^2)(2 + 2(\frac{1}{2})(-2))}{4(\frac{1}{2})^2(1)^2}$$

$$= \frac{+4 + 2(0)}{1} = +4$$

$\therefore$ At the point $(\frac{1}{2}, 1)$ the graph is decreasing and concave up

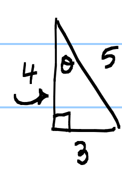


5)



$\frac{dx}{dt} = 0.8$ m/s

find $\frac{d\theta}{dt}$ when $x = 3$ m



when

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$\sin\theta = \frac{x}{5}$

$5\sin\theta = x$

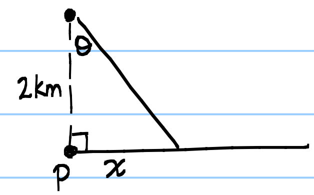
$5\cos\theta\frac{d\theta}{dt} = \frac{dx}{dt}$

$5\left(\frac{4}{5}\right)\frac{d\theta}{dt} = \frac{dx}{dt}$

$4\left(\frac{d\theta}{dt}\right) = 0.8$

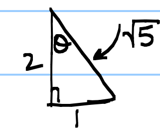
$$\frac{d\theta}{dt} = \frac{0.8}{4} = \boxed{0.2 \text{ rad/sec}}$$

6)



$\frac{d\theta}{dt} = 4(2\pi) = 8\pi$ rad/min

find $\frac{dx}{dt}$ when $x = 1$ km.



When

$\tan\theta = \frac{1}{2}$

$\sec\theta = \frac{\sqrt{5}}{2}$

$\sec^2\theta = \frac{5}{4}$

$\tan\theta = \frac{x}{2}$

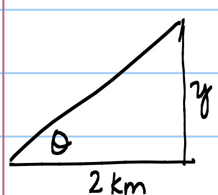
$2\tan\theta = x$

$2\sec^2\theta \frac{d\theta}{dt} = \frac{dx}{dt}$

$2\left(\frac{5}{4}\right)(8\pi) = \frac{dx}{dt}$

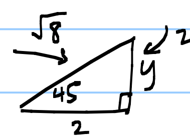
$\therefore \boxed{\frac{dx}{dt} = 20\pi \text{ km/min.}}$

7



$\frac{d\theta}{dt} = 3°/\text{sec}$ or $\frac{1}{60}\pi$ rad/sec

find $\frac{dy}{dt}$ when $\theta = 45°$



When $\theta = 45°$

$\sec\theta = \frac{\sqrt{8}}{2}$

$\sec^2\theta = \frac{8}{4} = 2$

$\tan\theta = \frac{y}{2}$

$2\tan\theta = y$

$2\sec^2\theta \frac{d\theta}{dt} = \frac{dy}{dt}$

$2(2)\left(\frac{1}{60}\pi\right) = \frac{dy}{dt}$

$\boxed{\frac{1}{15}\pi \text{ km/sec} = \frac{dy}{dt}}$

8) $m(x) = 2x\sin(2x)$

$m(\pi/8) = 2(\pi/8)\sin(\pi/4)$

$= 2(\pi/8)(\sqrt{2}/2) = \frac{\sqrt{2}\pi}{8} \doteq 0.555$

$m'(x) = 2\sin(2x) + 2x\cos(2x)(2)$

$m'(\pi/8) = 2\sin(\pi/4) + 4(\pi/8)\cos(\pi/4)$

$= 2(\sqrt{2}/2) + (\pi/2)(\sqrt{2}/2)$

$= \sqrt{2} + \frac{\sqrt{2}\pi}{4} \doteq 2.525$

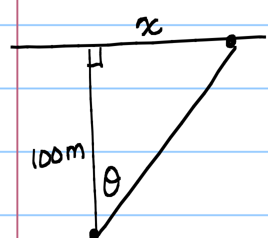
$\sqrt{2} + \frac{\sqrt{2}\pi}{4} = \frac{y - \frac{\sqrt{2}\pi}{8}}{x - \pi/8}$

Approximately:

$y = 2.525(x - 0.39) + 0.555$

$\left(\sqrt{2} + \frac{\sqrt{2}\pi}{4}\right)\left(x - \pi/8\right) + \frac{\sqrt{2}\pi}{8} = y$

9)

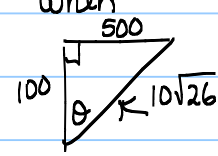


$\frac{d\theta}{dt} = -0.04$ rad/sec

find $\frac{dx}{dt}$ when $x = 500$m

$\tan\theta = \frac{x}{100}$

When



$\tan\theta = \frac{500}{100}$

$\sec\theta = \frac{10\sqrt{26}}{100} = \sqrt{26}$

$\sec^2\theta = 26$

$100\tan\theta = x$

$100\sec^2\theta\frac{d\theta}{dt} = \frac{dx}{dt}$

$100(26)(-0.04) = \frac{dx}{dt}$

$\frac{dx}{dt} = -104$ m/sec

$\frac{dx}{dt} = \left(-104\ m/s\right)\left(\frac{1\ km}{1000\ m}\right)\left(\frac{3600\ sec}{1\ hr}\right)$

$= -374.4$ km/hr.

↖ Speed of train is 104 m/s or 374.4 kph

10) $f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x$

$f''(x) = -\sin x - \cos x$

$f''(x) = 0$ when $-\sin x - \cos x = 0$

$-\sin x = \cos x$

$-\tan x = 1$

$\tan x = -1$

$\therefore$ When $x = 3\pi/4$